Applications of the Scalar Product

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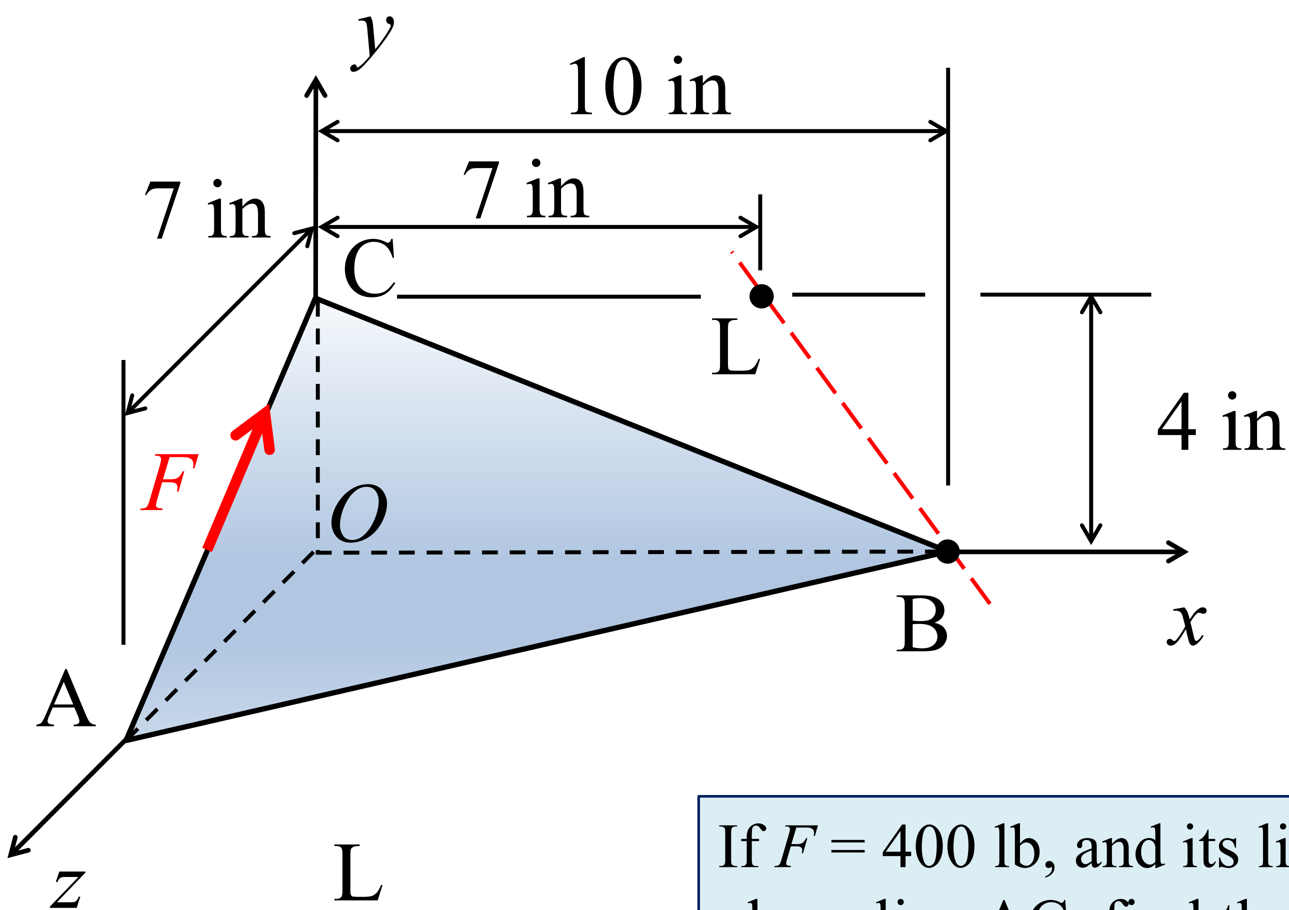
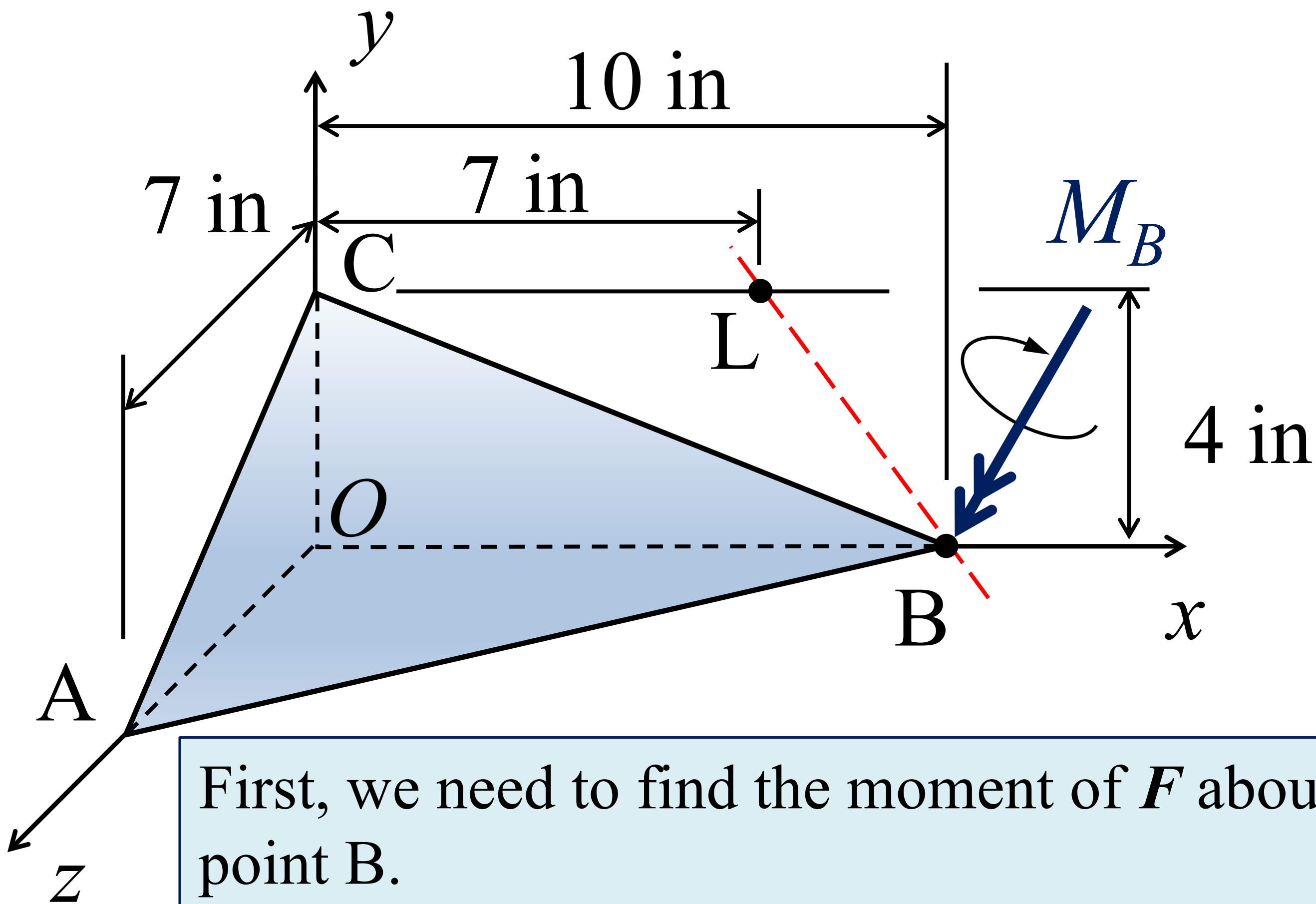
Moment of a Force About an Axis



If F = 400 lb, and its line-of-action lies along line AC, find the moment of the force about the axis defined by line BL that lies in the xy plane.

Moment of a Force About an Axis

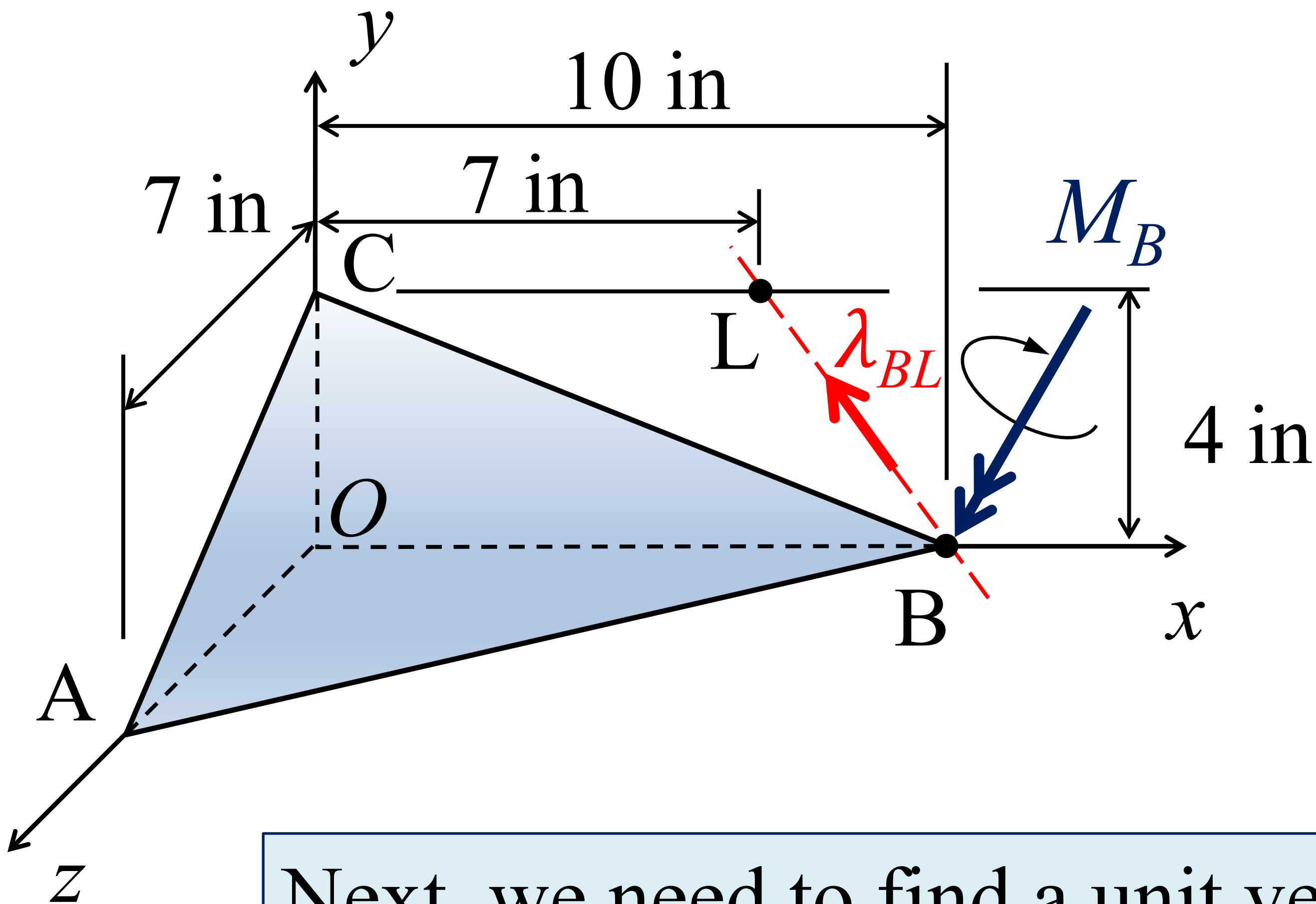


First, we need to find the moment of ***F*** about point B.

Recall that we found $\boldsymbol{M_B}$ in a previous example.

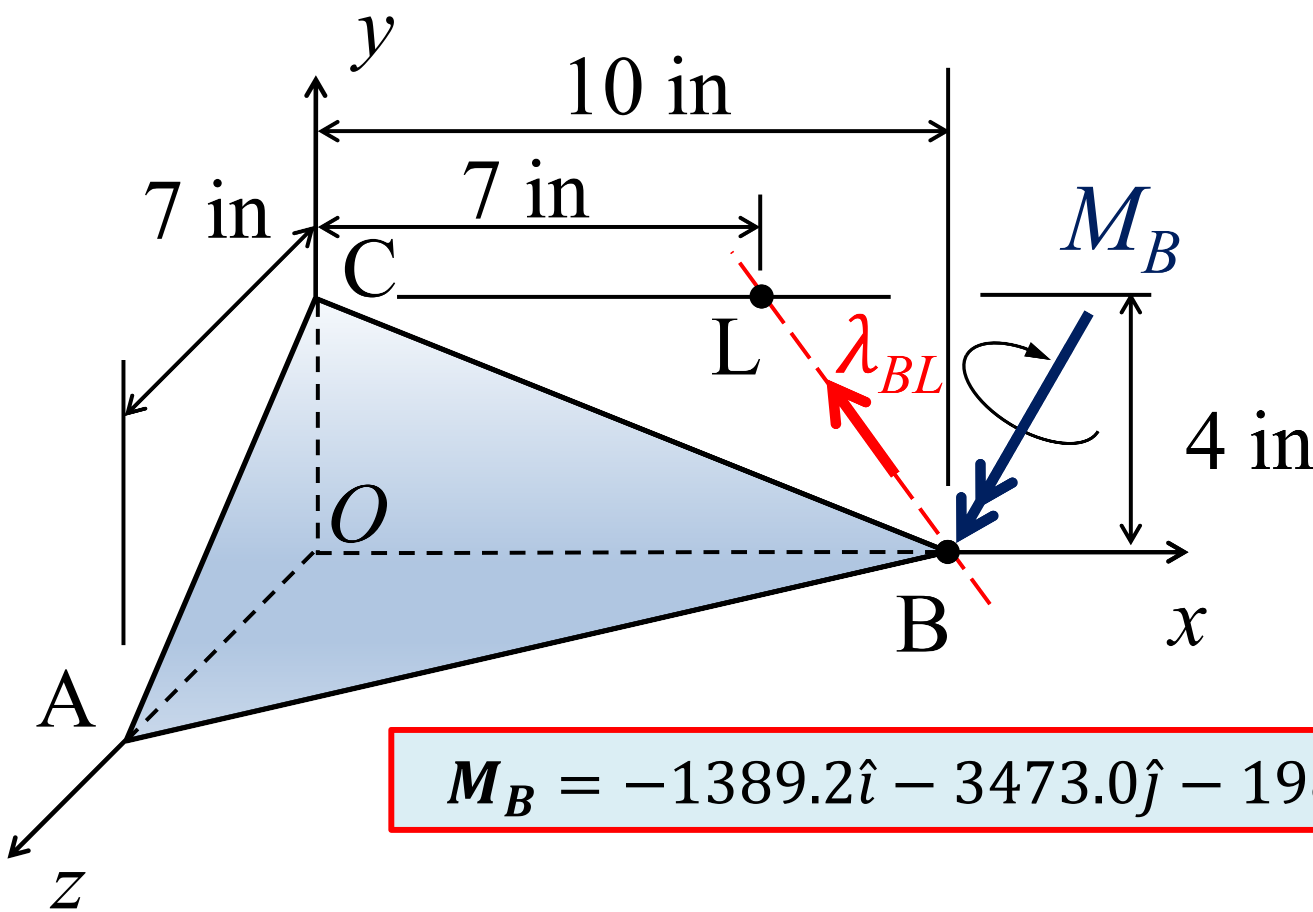
$$\boldsymbol{M_B} = -1389.2\hat{\imath} - 3473.0\hat{\jmath} - 1984.6\hat{k} \quad \text{lb-in}$$

Moment of a Force About an Axis



Next, we need to find a unit vector, $\boldsymbol{\lambda_{BL}}$, along the line BL.

Moment of a Force About an Axis

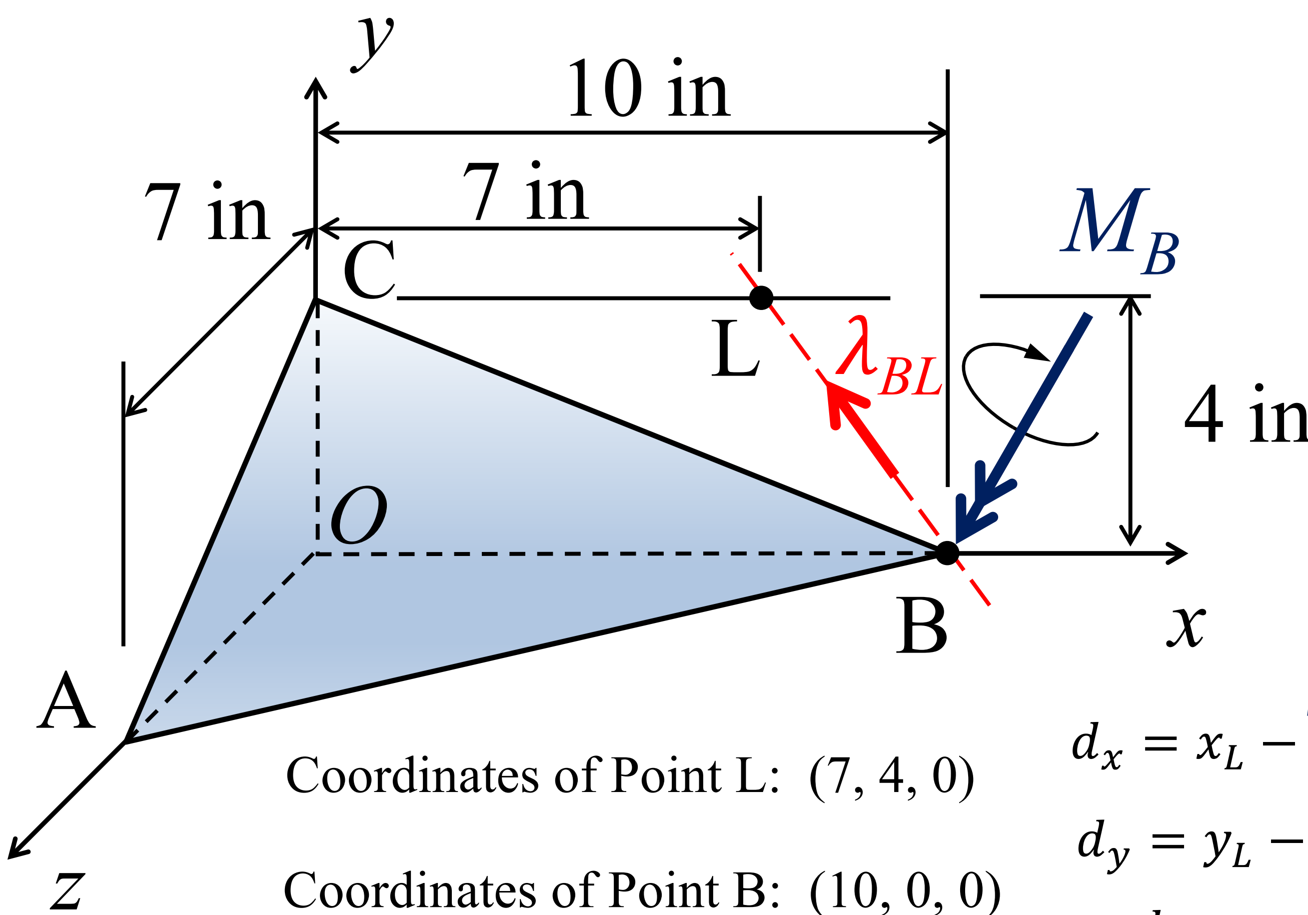


$$\boldsymbol{M_B} = -1389.2\hat{\imath} - 3473.0\hat{\jmath} - 1984.6\hat{k} \quad \text{lb−in}$$

$$M_{BL} = \boldsymbol{M_B} \cdot \lambda_{BL}$$

Since $\boldsymbol{\lambda_{BL}}$ is a unit vector, the scalar product of $\boldsymbol{M_B}$ and $\boldsymbol{\lambda_{BL}}$ will be the projection of $\boldsymbol{M_B}$ onto the axis BL.

Find λ_{BL} in Cartesian Vector Form



Coordinates of Point L: (7, 4, 0)

Coordinates of Point B: (10, 0, 0)

Tip minus Tail

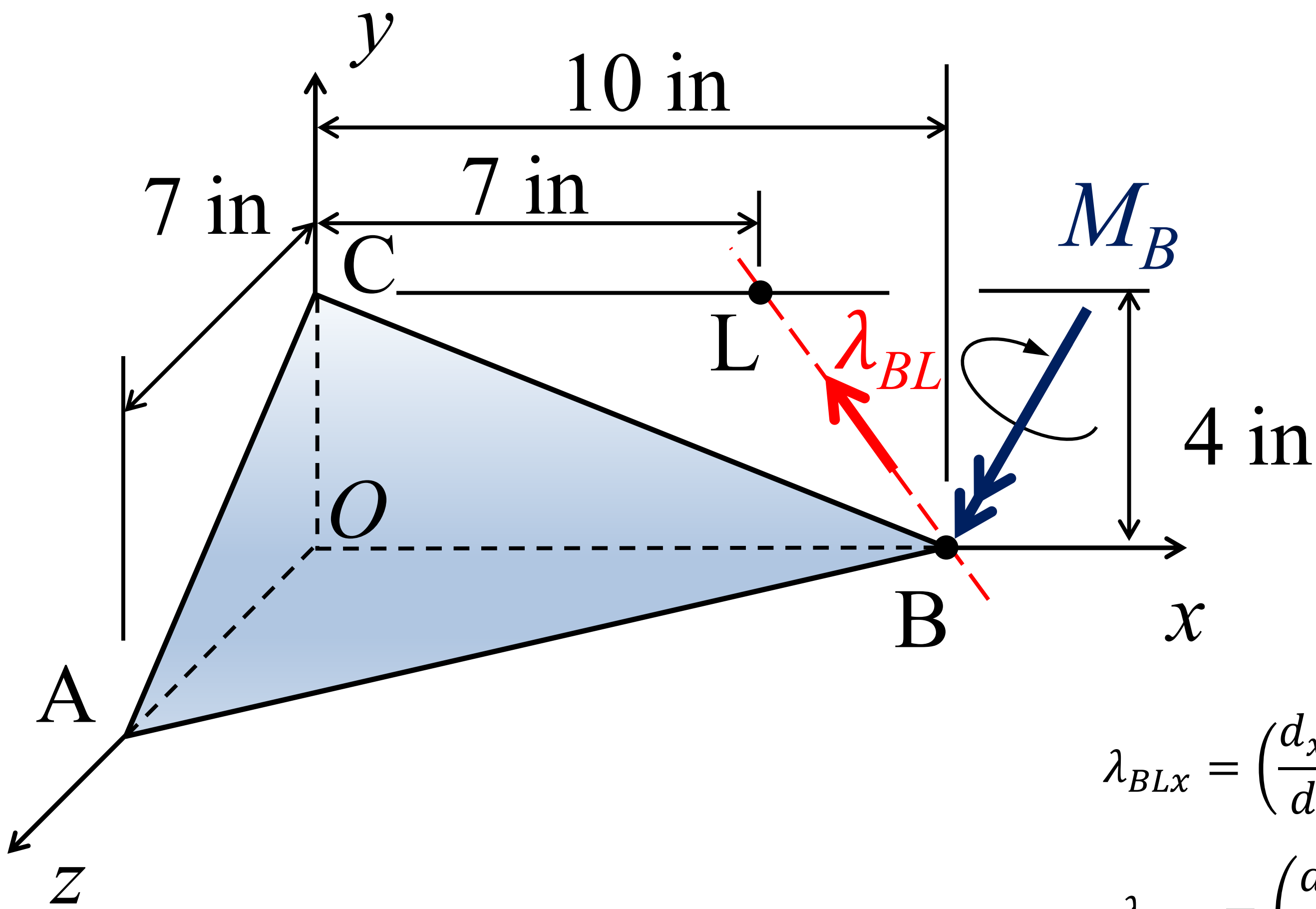
$$d_x = x_L - x_B = 7 - 10 = -3 \text{ in}$$

$$d_y = y_L - y_B = 4 - 0 = 4 \text{ in}$$

$$d_z = z_L - z_B = 0 - 0 = 0$$

$$d = \sqrt{(-3)^2 + (4)^2 + (0)^2} = 5.0 \text{ in}$$

Find λ_{BL} in Cartesian Vector Form



$$d = \sqrt{(-3)^2 + (4)^2 + (0)^2} = 5.0 \text{ in}$$

$$\lambda_{BLx} = \left(\frac{d_x}{d}\right) = \left(\frac{-3}{5}\right) = -0.6$$

$$\lambda_{BLy} = \left(\frac{d_y}{d}\right) = \left(\frac{4}{5}\right) = 0.8$$

$$\lambda_{BLz} = \left(\frac{d_y}{d}\right) = \left(\frac{0}{5}\right) = 0$$

$$\boldsymbol{\lambda_{BL}} = -0.6\hat{\imath} + 0.8\hat{\jmath}$$

Moment of a Force About an Axis

$$\boldsymbol{M_B} = -1389.2\hat{\imath} - 3473.0\hat{\jmath} - 1984.6\hat{k} \ \ \text{lb−in}$$

$$\boldsymbol{\lambda_{BL}} = -0.6\hat{\imath} + 0.8\hat{\jmath}$$

$$M_{BL} = \boldsymbol{M_B} \cdot \boldsymbol{\lambda_{BL}}$$

$$M_{BL} = M_{Bx}\lambda_{BLx} + M_{By}\lambda_{BLy} + M_{Bz}\lambda_{BLz}$$

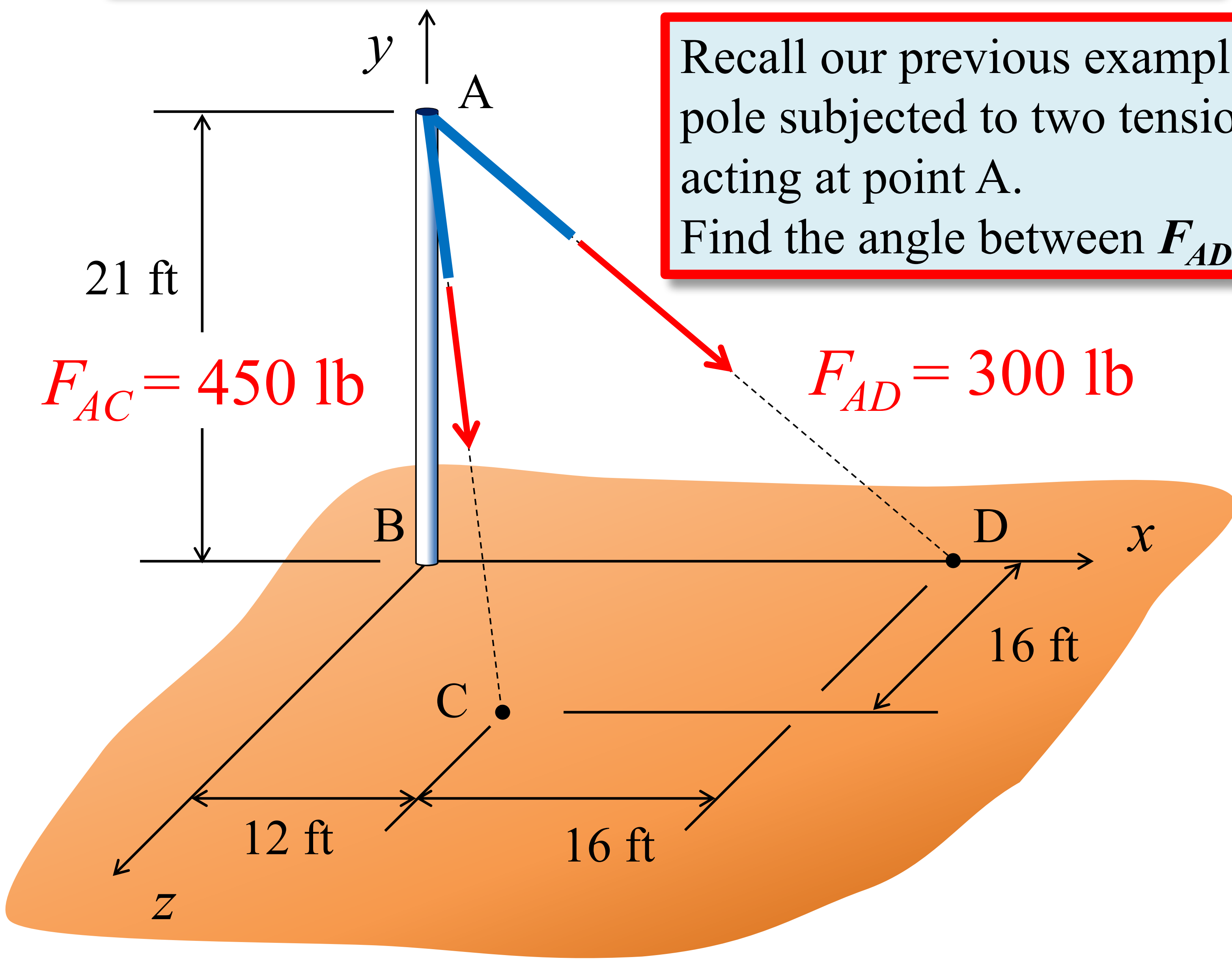
$$M_{BL} = (-1389.2)(-0.6) + (-3473.0)(0.8) + (-1984.6)(0)$$

$$M_{BL} = -1944.9 \text{ lb−in}$$

Negative sign indicates that the projection is in the opposite direction of the sense of $\boldsymbol{\lambda_{BL}}$

Angle Between Two Vectors in Space

Recall our previous example with a pole subjected to two tension forces acting at point A.
Find the angle between $\boldsymbol{F}_{AD}$ and $\boldsymbol{F}_{AC}$



Recall the Definition of the Scalar Product

From the definition of the scalar Product:

$$\boldsymbol{P} \cdot \boldsymbol{Q} = PQ\cos\theta$$

$$\cos\theta = \frac{\boldsymbol{P} \cdot \boldsymbol{Q}}{PQ}$$

We need to find the magnitude of each vector and their scalar product

For our problem:

$$\cos\theta = \frac{\boldsymbol{F}_{\boldsymbol{AC}} \cdot \boldsymbol{F}_{\boldsymbol{AD}}}{F_{AC}F_{AD}}$$

Vectors in Cartesian Vector Form

Recall that in a previous example, we found:

$$\boldsymbol{F_{AC}} = 186.21\hat{\imath} - 325.86\hat{\jmath} + 248.28\hat{k} \text{ lb}$$

$$\boldsymbol{F_{AD}} = 240\hat{\imath} - 180\hat{\jmath} \text{ lb}$$

The magnitudes of each vector are given:

$$F_{AC} = 450 \text{ lb}$$

$$F_{AD} = 300 \text{ lb}$$

Scalar Product of Two Vectors in Cartesian Vector Form

$$\boldsymbol{P} \cdot \boldsymbol{Q} = P_x Q_x + P_y Q_y + P_z Q_z$$

$$\boldsymbol{F}_{\boldsymbol{AC}} \cdot \boldsymbol{F}_{\boldsymbol{AD}} = F_{ACx} F_{ADx} + F_{ACy} F_{ADy} + F_{ACz} F_{ADz}$$

$$\boldsymbol{F}_{\boldsymbol{AC}} = 186.21\hat{\imath} - 325.86\hat{\jmath} + 248.28\hat{k} \text{ lb}$$

$$\boldsymbol{F}_{\boldsymbol{AD}} = 240\hat{\imath} - 180\hat{\jmath} \text{ lb}$$

$$\boldsymbol{F}_{\boldsymbol{AC}} \cdot \boldsymbol{F}_{\boldsymbol{AD}} = (186.21)(240) + (-325.86)(-180) + (248.28)(0)$$

$$\boldsymbol{F}_{\boldsymbol{AC}} \cdot \boldsymbol{F}_{\boldsymbol{AD}} = 103{,}344.9 \text{ lb}^2$$

Angle Between the Two Vectors

$$\cos\theta = \frac{\boldsymbol{F}_{AC} \cdot \boldsymbol{F}_{AD}}{F_{AC} F_{AD}}$$

$$\boldsymbol{F}_{AC} \cdot \boldsymbol{F}_{AD} = 103{,}344.9 \text{ lb}^2$$

$$F_{AC} = 450 \text{ lb}$$

$$F_{AD} = 300 \text{ lb}$$

$$\cos\theta == \frac{103{,}344.9 \text{ lb}^2}{(450 \text{ lb})(300 \text{ lb})} = 0.7655$$

$$\theta = \cos^{-1}(0.7655) = 40.0°$$

Angle Between Two Vectors in Space